Title: Maximum bound principles for a class of semilinear parabolic equations

and exponential time differencing schemes

by

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The ubiquity of semilinear parabolic equations has been illustrated in their numerous applications ranging from physics, biology, to materials and social sciences. In this paper, we consider a practically desirable property for a class of semilinear parabolic equations of the abstract form $u_t=\hL u+f[u]$ with $\hL$ being a linear dissipative operator and f being a nonlinear operator in space, namely a time-invariant maximum bound principle, in the sense that the time-dependent solution u preserves for all time a uniform pointwise bound imposed by its initial and boundary conditions.

We first study an analytical framework for some sufficient conditions on $\hL$ and f that lead to such a maximum bound principle for the time-continuous dynamic system of infinite or finite dimensions. Then, we design suitable exponential time differencing approach with a properly chosen generator of the semigroup to develop first- and second-order accurate temporal discretization schemes that satisfy the maximum bound principle unconditionally in the time-discrete setting. Error estimates of the proposed schemes are derived along with their energy stability. Extensions to vector- and matrix-valued systems are also discussed. We demonstrate that the abstract framework and analysis techniques developed here offer an effective and unified approach to study the maximum bound principle of the abstract evolution equation that cover a wide variety of well-known models and their numerical discretization schemes.

Some numerical experiments are also carried out to verify the theoretical results.