Recent advances on structure preserving schemes for complex nonlinear systems

by

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Abstract: Many complex nonlinear systems have intrinsic structures such as energy dissipation or conservation, and/or positivity/maximum principle preserving. It is desirable, sometimes necessary, to preserve these structures in a numerical scheme.

I will first present the scalar auxiliary variable (SAV) approach to deal with nonlinear terms in a large class of dissipative/conservative systems. The SAV approach is not restricted to specific forms of the nonlinear part of the free energy or Hamiltonian. It leads to linear and unconditionally energy stable schemes which only require solving decoupled linear equations with constant coefficients. Hence, these schemes are extremely efficient and very accurate when combined with higher-order BDF or diagonally implicit Runge-Kutta schemes.

However, the SAV approach will not preserve positivity or maximum principle. I will then present a strategy to construct efficient energy stable and positivity preserving schemes for certain nonlinear evolution systems, such as the Poisson-Nernst-Planck (PNP) equation and Keller-Segel equation, whose solutions remain to be positive.